MTH 237-60

Extra credit 4. The unreasonable effectiveness of matrix multiplication (1.5 pt)

Let V have ordered basis $\mathscr{B} = \{\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_m\}$, W have ordered basis $\mathscr{C} = \{\mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_n\}$, and X have ordered basis $\mathscr{D} = \{\mathbf{d}_1, \mathbf{d}_2, \ldots, \mathbf{d}_k\}$. Let $T : V \to W$ be the linear transformation defined by

$$T(\mathbf{b}_j) = \sum_{i=1}^{n} a_{i,j}\mathbf{c}_i, \qquad a_{i,j} \in \mathbf{R}$$

and let $U : W \to X$ be the linear transformation defined by

$$U(\mathbf{c}_j) = \sum_{\ell=1}^{k} b_{\ell,j}\mathbf{d}_\ell, \qquad b_{\ell,j} \in \mathbf{R}.$$

Compute $A = [T]_{\mathscr{B},\mathscr{C}}$, $B = [U]_{\mathscr{C},\mathscr{D}}$, and verify that the j-th column of BA is $[UT(\mathbf{b}_j)]_{\mathscr{D}}$ as expected.

Hint. This question will look insanely hard at first. Even when you get the hang of things, it is still kinda hard (hence its higher bounty). When solving highly abstract problems such as these, it is useful to consider small values of m, n, and k, doing the same problem for easy numbers and then applying the patterns you see to the general case.