

### **MTH 237-60**

*Final study guide.* The final will take place on Tuesday 6/28 with a review session on Monday 6/27. The final will be comprised of 70% questions about the basics (definitions, procedures, *etc.*) and 30% “application” questions not unlike Homework 4. You should know:

THE CONTENT OF THE MIDTERM BY HEART: If there is a question on the midterm so important that your knowledge of it directly influences whether or not you pass the course, do not be surprised to see that question reappear verbatim on the final.

VECTOR SPACES: What is a vector space? (Seriously, do not skip over learning that definition.) Can you verify that a set is a vector space? What is a linear combination? What does it mean for a set of vectors to be linearly independent? What is a basis? Can you find or verify a basis for a given vector space? What is a subspace and can you verify that a given subset of a vector space is one?

LINEAR TRANSFORMATIONS: What is a linear transformation? Can you verify that a function is linear? What are the image and kernel of a linear transformation? Can you compute these for a given linear transformation? What is an isomorphism? How can you tell if a linear transformation is an isomorphism? What is the rank-nullity theorem and what does it mean?

COORDINATIZATION: Give the isomorphism between a vector space  $V$  of dimension  $n$  and  $\mathbf{R}^n$ . If  $V$  and  $W$  are vector spaces of dimension  $m$  and  $n$  respectively, give the isomorphism between  $\text{Hom}(V, W)$  and  $M_{m \times n}(\mathbf{R})$ . Understand the relationship between linear composition and matrix multiplication.

THE EIGENVALUE PROBLEM: Given a linear operator  $T$  from a vector space  $V$  to itself, what are the eigenvalues and their associated eigenvectors of  $T$ ? What is the characteristic polynomial of  $T$ ? What does it mean for a matrix to be diagonalizable and what does it mean when it's not? Be able to interpret these ideas geometrically.