MTH 237-60 **Name:**

Homework 1. Gauss-Jordan elimination

Write neatly and completely. Work in groups if you like, but your submission must be your own. Feel free to e-mail me or come to office hours with questions.

You may remember that an linear equation in two variables, usually given $y = mx + b$ but $Ax + By = C$ is fine, represents a line in the Cartesian coordinate plane. The intersection of two lines is a single point, which is why given two such **linearly independent equations** (*i.e.* the lines they represent are not parallel) one can find a unique point solution. A linear equation in three variables represents a plane in space.[7] The intersection of how many independent (non-redundant) planes is a single point?

What do you think the intersection of two independent planes is?

[7]Linear equations in four or more variables, whose images are impossible to visualize, are called **hyperplanes**.

Let's test your answer to the preceding question with an example. Consider the intersection of the following planes:

$$3x + y - 2z = 5$$
$$7x - y = 2$$

Set up and row-reduce an augmented matrix that represents this system. For full credit, show every step in the row reduction process. Feel free to use Octave to calculate each step.

Write your solution in the form of a column vector (*c.f.* the second example in I.1.).

Your solution should be in the form

$$\begin{bmatrix} A + Bz \\ C + Dz \\ E + Fz \end{bmatrix}$$

where A, B, C, D, E, and F are real numbers. Using the operations on $\mathbf{R}^3$ (from calculus and reviewed in class if necessary), work this into something of the form

$$\begin{bmatrix} A \\ C \\ E \end{bmatrix} + z \begin{bmatrix} B \\ D \\ F \end{bmatrix} = \mathbf{r} + z\mathbf{v}.$$

Geometrically, this is interpreted as follows. We follow the vector $\mathbf{r}$ out A units east,[8] C units north, and E units up from the origin. This gives us the "starting point" of our shape (which yes, good guess, is a line.) From the point (A, C, E), we travel z units in the direction of the vector $\mathbf{v}$, which is B units east, D units north, and F units up. The parameter z can be any real number, as we were unable to solve the system exactly (we didn't have enough equations).

Draw a picture of the resulting line as best as you can (your artistic ability does not influence your grade).

[8]For lack of better terminology, we will say the positive x axis is east, the positive y axis is north, and the positive z axis is up.