MTH 237-60 **Name:**

Homework 3. Vector space basics. Due Tuesday 6/21 at the beginning of class.

1. Let n be any positive integer and

$$\mathbf{R}^n = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} : x_1, x_2, \ldots, x_n \in \mathbf{R} \right\}.$$

Treat addition and scalar multiplication on $\mathbf{R}^n$ as if it were the set of $n \times 1$ real matrices. In other words, if $\mathbf{x} = [x_1, x_2, \ldots, x_n]^t$, $\mathbf{y} = [y_1, y_2, \ldots, y_n]^t$, $(\mathbf{x} + \mathbf{y})_i$ denotes the i-th coordinate of $\mathbf{x} + \mathbf{y}$, and $\lambda \in \mathbf{R}$, then

$$(\mathbf{x} + \mathbf{y})_i = x_i + y_i$$

and

$$(\lambda \mathbf{x})_i = \lambda x_i.$$

1.a. Show that $\mathbf{R}^n$ is a vector space.

1.a., continued:

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1.b. Show that the set $\{\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_n\} \subset \mathbf{R}^n$ where $\mathbf{e}_i$ is the column vector with a 1 in its i-th coordinate and a 0 in all other coordinates is a basis for $\mathbf{R}^n$.