**MTH 237-60** **Name:**

*Homework 4. Projection maps.* Due Tuesday 6/28 before the exam.

Let $\mathbf{x} = [2, 0, 1]^t$ and $\mathbf{y} = [0, -3, 1]^t$ in the vector space $\mathbf{R}^3$. Fix a third vector $\mathbf{z}$ that is linearly independent of $\mathbf{x}$ and $\mathbf{y}$. Then, any vector $\mathbf{v}$ in $\mathbf{R}^3$ can be written

$$\mathbf{v} = \alpha_1\mathbf{x} + \alpha_2\mathbf{y} + \alpha_3\mathbf{z}$$

for unique scalars $\alpha_1, \alpha_2, \alpha_3$.

Let $\pi : \mathbf{R}^3 \to \mathbf{R}^3$ be the map defined by

$$\pi(\alpha_1\mathbf{x} + \alpha_2\mathbf{y} + \alpha_3\mathbf{z}) = \alpha_1\mathbf{x} + \alpha_2\mathbf{y}.$$

The map $\pi$ is called a projection map since $\pi^2(\mathbf{v}) = \pi(\pi(\mathbf{v})) = \pi(\mathbf{v})$ for all $\mathbf{v} \in \mathbf{R}^3$.

1. Verify that $\pi$ is linear.

2. Compute im $\pi$ and verify that it is a subspace of $\mathbf{R}^3$. Interpret it geometrically in relation to $P(\mathbf{x}, \mathbf{y})$, the plane spanned by $\mathbf{x}$ and $\mathbf{y}$. What is the rank of $\pi$?

3. Compute ker $\pi$ and verify that it is a subspace of $\mathbf{R}^3$. Interpret it geometrically in relation to $P(\mathbf{x}, \mathbf{y})$. What is the nullity of $\pi$?

4. Compute the matrix for $\pi$ according to the standard basis $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$.

5. Give the eigenvalues for $\pi$, their corresponding eigenvectors, and interpret them geometrically. If possible, diagonalize the matrix you found in (4).