MTH 237-60

Midterm study guide. The midterm will take place on Thursday 6/9 with a review session / catch-up day (if necessary) on Wednesday 6/8. There will be some questions about the basics, one large determinant to calculate, and one application problem. You should know:

THE BASICS OF USING MATRICES TO SOLVE LINEAR SYSTEMS: when is an equation linear? When does a system have one, infinitely many, or no solutions? How do you use the reduced row echelon form of an augmented matrix to solve a system? How do you use matrix inversion to solve a system, and when is that possible? What is Cramer's rule and how does it work?

MATRIX ALGEBRA: when can you add matrices and how do you do it? Scalar multiplication? Matrix multiplication? Transposition? If the inverse of a matrix exists, what is it and how is it found? What are the rules for these operations—*e.g.* is multiplication commutative? What is $(AB)^t$ for compatible matrices A and B?

DETERMINANTS: loosely speaking, what is a determinant? How do you find it? How do you use row reduction to simplify a determinant and what effect does each move have on the result?

APPLICATIONS: what is a stochastic matrix? Given a stochastic matrix and a distribution of genotypes in a population, how can we use matrix multiplication to see what future generations will look like? How can we use invertible matrices to encrypt messages?

MATHEMATICS: know how to communicate mathematical ideas in the abstract, *i.e.* in full generality. Be able to talk about a real number $\lambda \in \mathbf{R}$ without needing to give it a value. Be able to understand the sentences:

$$(AB)_{i,j} = [\text{row } i \text{ of } A] \cdot [\text{column } j \text{ of } B]$$

and

Let $A \in M_{n\times n}(\mathbf{R})$ *and* $1 \leq i \leq n$. *Then*

$$\det A = \sum_{j=1}^{n} (-1)^{i+j} A_{i,j} C_{i,j}.$$

(Also: if you don't use the dot product to describe matrix multiplication, make sure you have something *way* better. "Filling the bucket" is not better.)